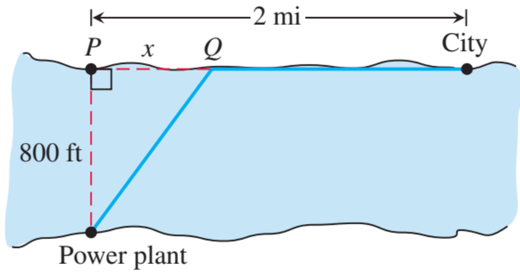
Exercise 76

Industrial costs A power plant sits next to a river where the river is 800 ft wide. Laying a new cable from the plant to a location in the city 2 mi downstream on the opposite side costs \$180 per foot across the river and \$100 per foot along the land.



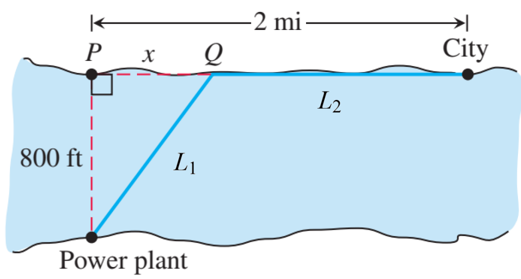
NOT TO SCALE

a. Suppose that the cable goes from the plant to a point Q on the opposite side that is x ft from the point P directly opposite the plant. Write a function $C(x)$ that gives the cost of laying the cable in terms of the distance x.

b. Generate a table of values to determine whether the least expensive location for point Q is less than 2000 ft or greater than 2000 ft from point P.

Solution

Label the length of the hypotenuse as L_1 and the length from Q to the city as L_2.



The sides of a right triangle are related by the Pythagorean theorem.

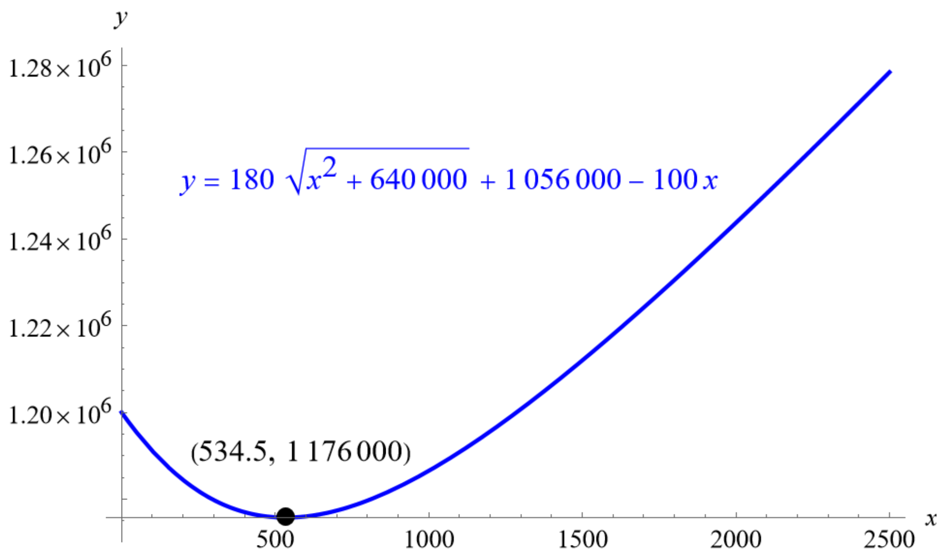
$$800^2 + x^2 = L_1^2$$

$$L_1 = \sqrt{x^2 + 640\,000}$$

The cost of laying the cable as a function of x is then (note 1 mile is 5280 feet)

$$\begin{aligned} C(x) &= 180L_1(x) + 100L_2(x) \\ &= 180\sqrt{x^2 + 640\,000} + 100(2 \cdot 5280 - x) \\ &= 180\sqrt{x^2 + 640\,000} + 1\,056\,000 - 100x. \end{aligned}$$

Below is a graph of the cost function versus x.



It's least expensive to lay cable when P is about 534 feet from Q, which is less than 2000 feet. The minimum cost is about \$1,176,000.